The Spectral Relation between the Cube-Connected Cycles and the Shuffle-Exchange Network

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[1]Pattern Recognition Lab (CS 5)
[2]Hardware-Software Co-Design (CS 12),
[3]Artificial Intelligence (CS 8)

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Spectra of Networks

- Spectral set ([redacted]) contains information on
  - Network throughput
  - Fault-tolerance
  - …
- Known spectra:
  - Linear array
  - Cycle
  - Hypercube
  - Butterfly
  - De Bruijn

[redacted]

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[redacted]

- Known spectra:
  - Linear array
  - Cycle
  - Hypercube
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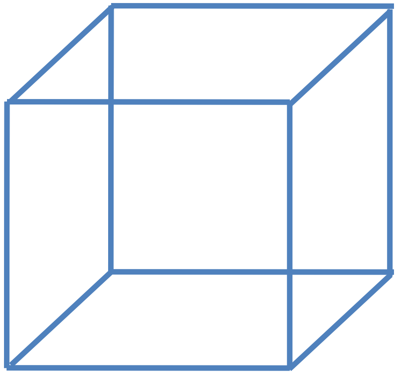
  - De Bruijn
- $\mathrm{SpS}(\mathrm{CCC}(d)) = ?$
- $\mathrm{SpS}(\mathrm{SE}(d)) = ?$

Parallel Computations on the Hypercube

- [redacted]-dimensional Hypercube: a popular architecture for parallel computations

- Versatile connection structure for many algorithms
  - Finite differences
  - Spanning trees
  - Connected components
  - Sorting networks
  - Nearest neighbor search
  - Chinese remaindering
  - …

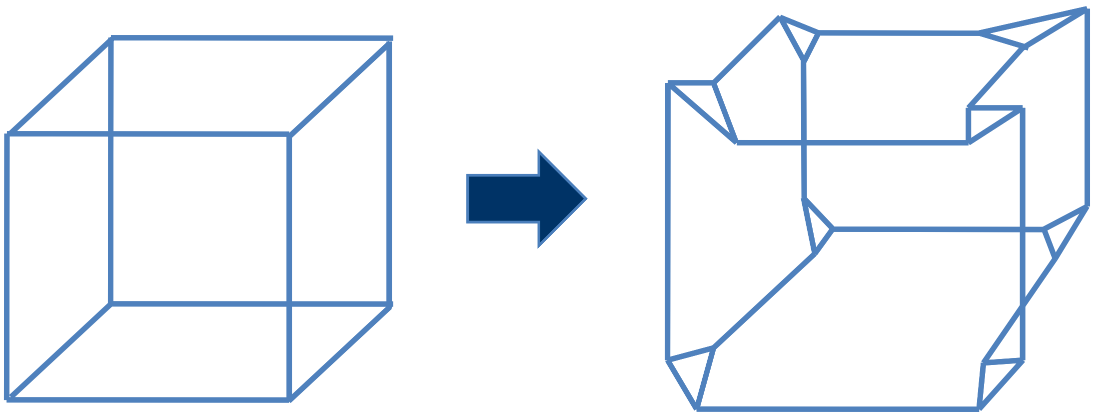


[redacted]

- Drawback: degree [redacted] in every node

Hypercubic: Cube-Connected Cycles Network

- [redacted]-dimensional Cube-Connected Cycles [redacted]: [redacted] nodes
- Starting from a hypercube, every node is replaced by a cycle



[redacted]

- Runs hypercube algorithms with constant slowdown
- Constant degree [redacted]

Hypercubic: Shuffle-Exchange Network

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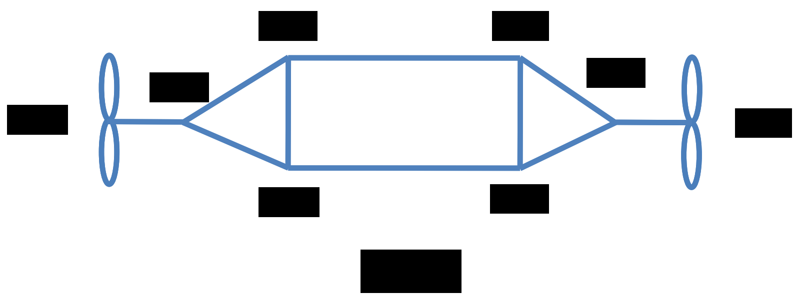
- Connections according to bit pattern:
  - Cyclic left- or right shifts
  - Flip (exchange) of the last bit



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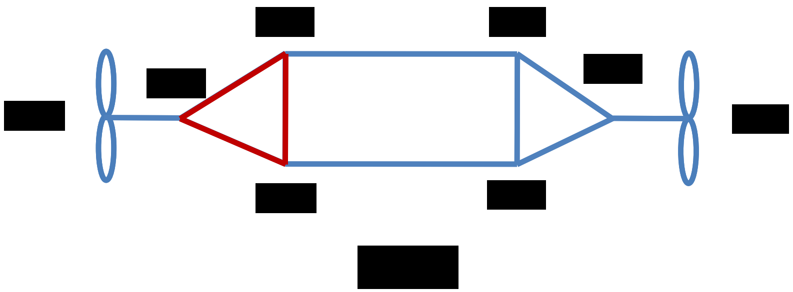

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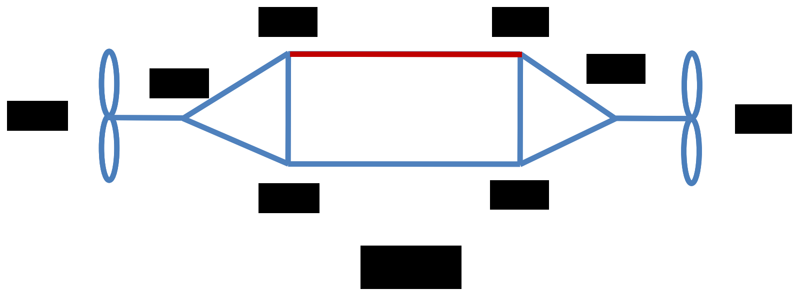

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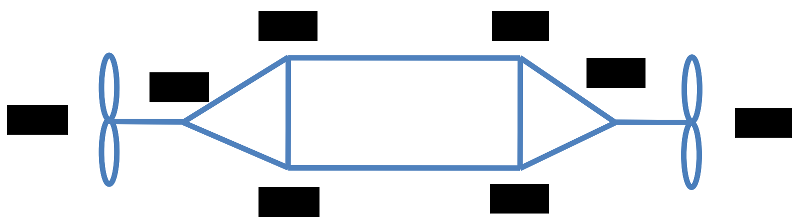

[redacted]

- Runs hypercube algorithms with constant slowdown
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Mathematical Tools for Network Characterization

- Network = Graph (represented by its adjacency matrix)

- Spectral graph theory:

  Examine relationship between
  the network and its Eigenvalues/Eigenvectors

- A small example:

Adjacency:

[redacted]

Eigenvalues:

[redacted]

Average degree:

[redacted]

Bounds from Graph Spectra

- Isoperimetric number
- Expansion property
- Routing number
- Chromatic number
- Independence number
- Bisection width

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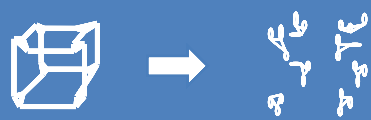
Message destinations in [redacted]

Number nodes

2nd largest Eigenvalue

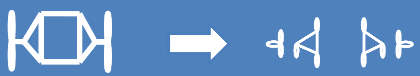
Spectral Relation of CCC and SE

Decomposing CCC



- Graph transform
- Spectrum is identical

Decomposing SE



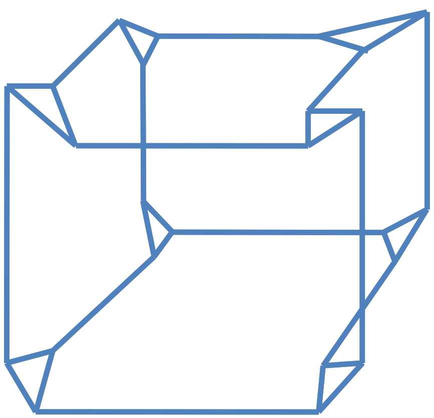
- Similarity transform
- Spectrum is identical

Relation of CCC and SE

- Matching of subgraphs
- If d odd: $\mathrm{SpS}(\mathrm{CCC}(d)) = \mathrm{SpS}(\mathrm{SE}(d))$
- If d even: $\mathrm{SpS}(\mathrm{CCC}(d)) \supset \mathrm{SpS}(\mathrm{SE}(d))$

Decomposing CCC

- Spectrum-preserving graph editing



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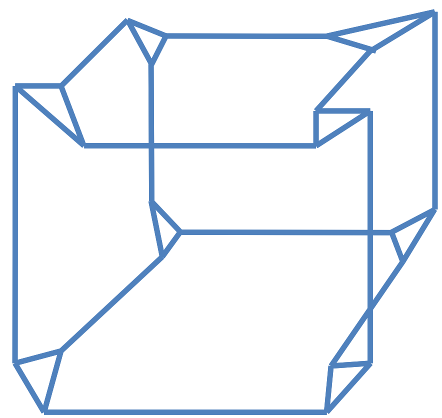

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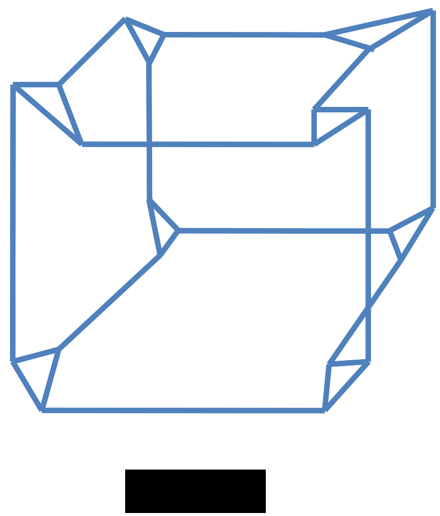

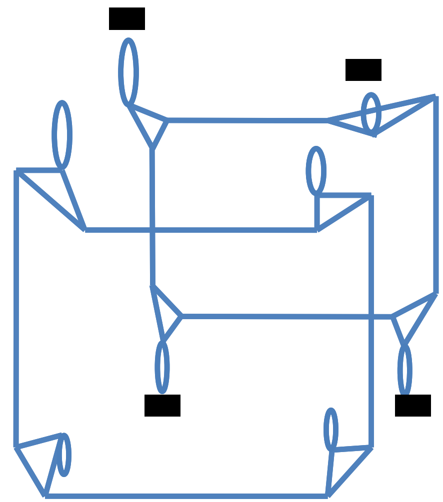

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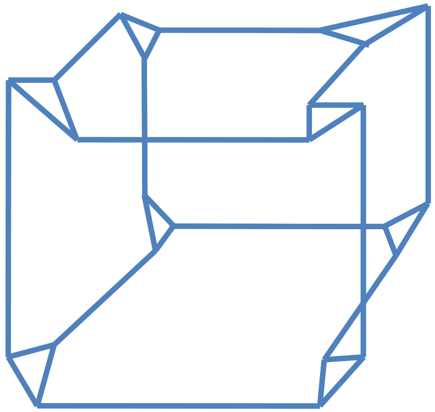

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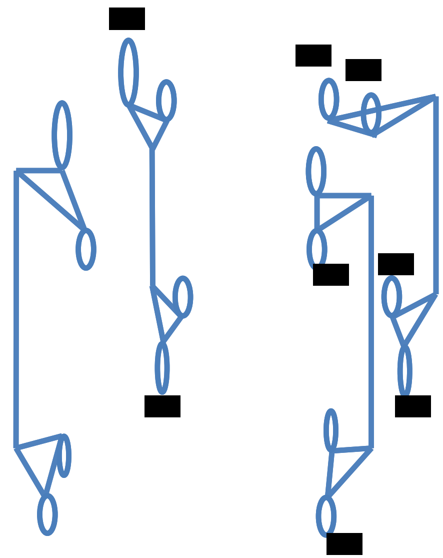

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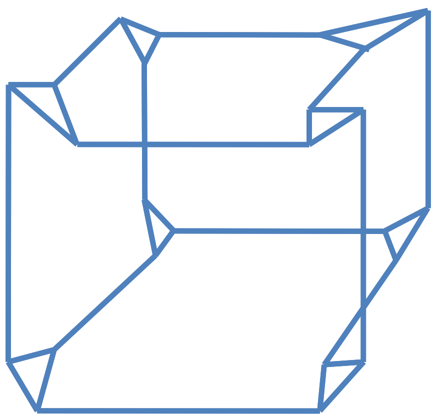

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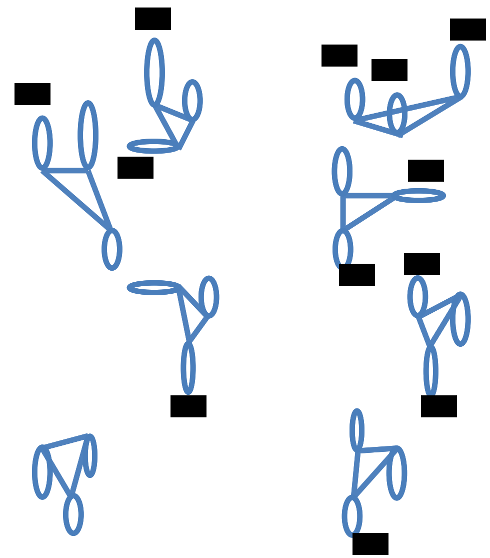

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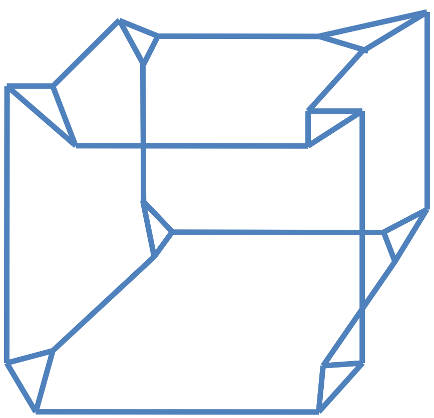

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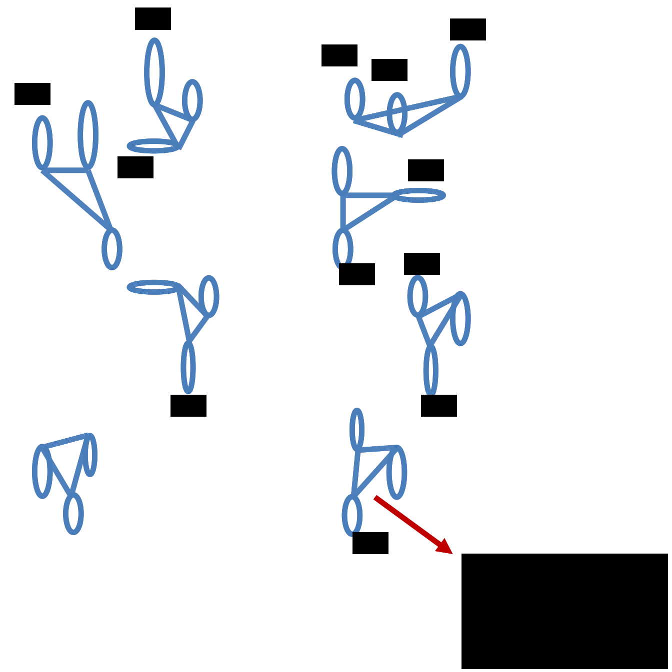

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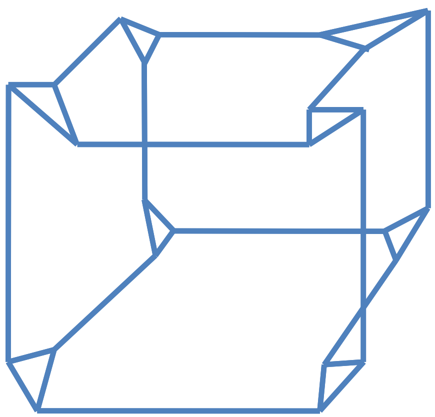

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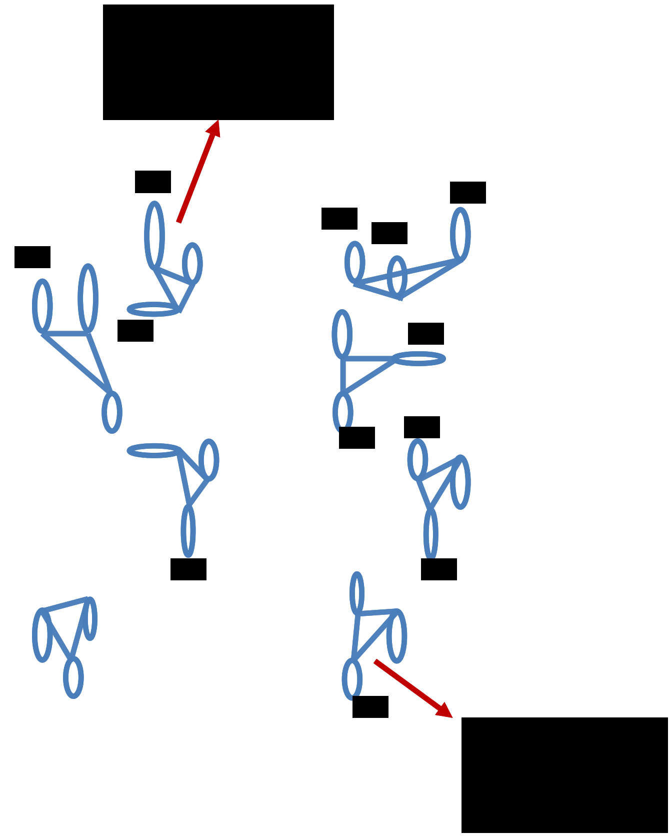

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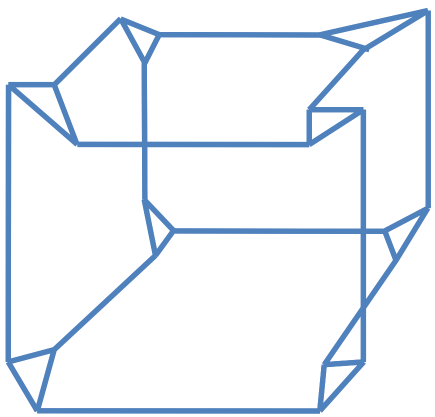

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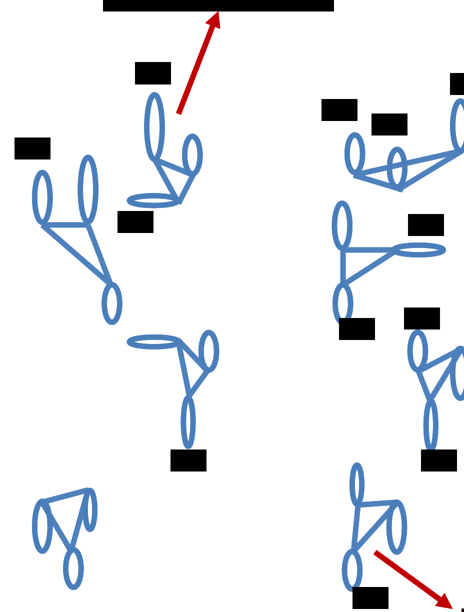

[redacted]

disconnected
subgraphs

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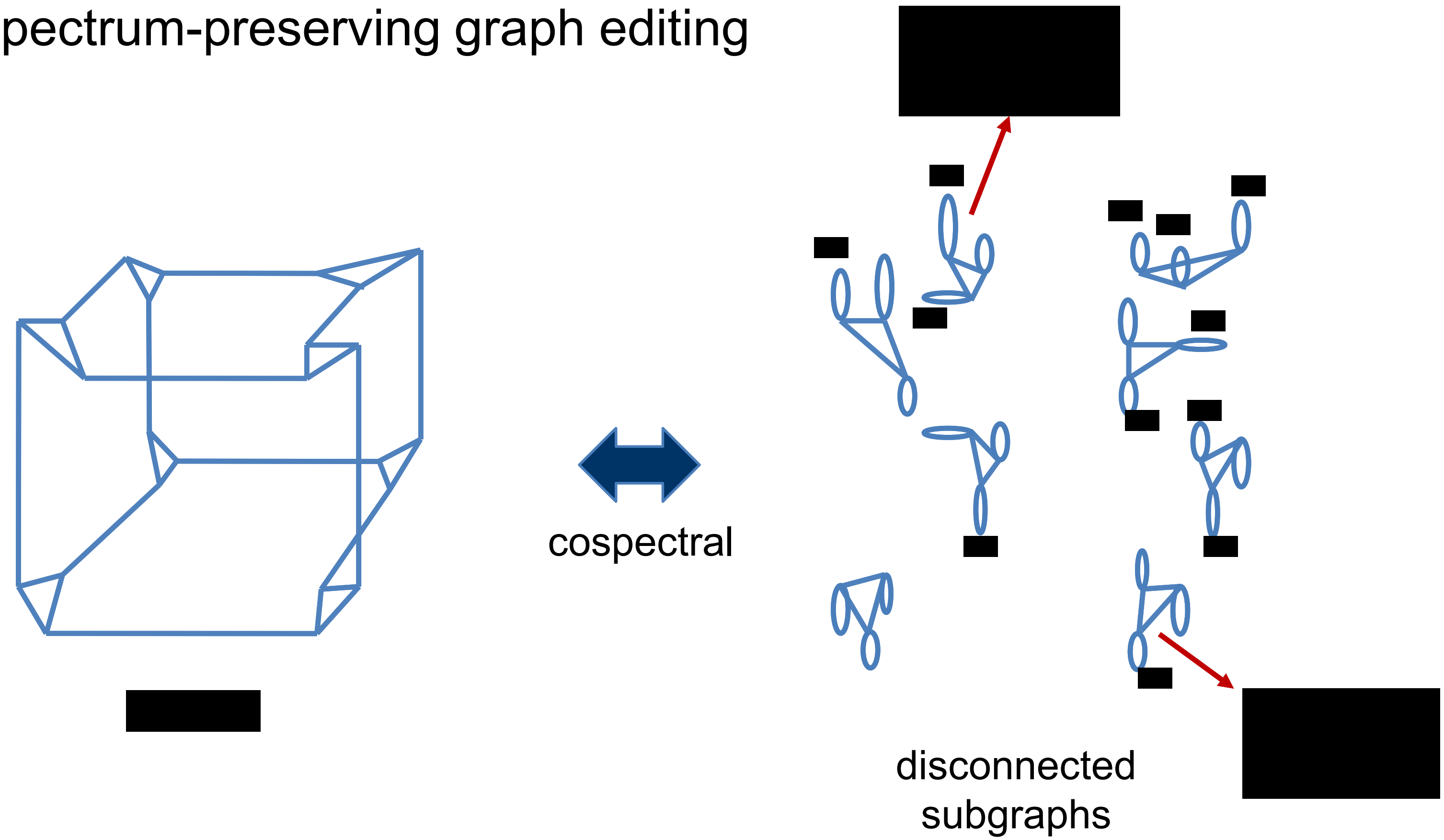
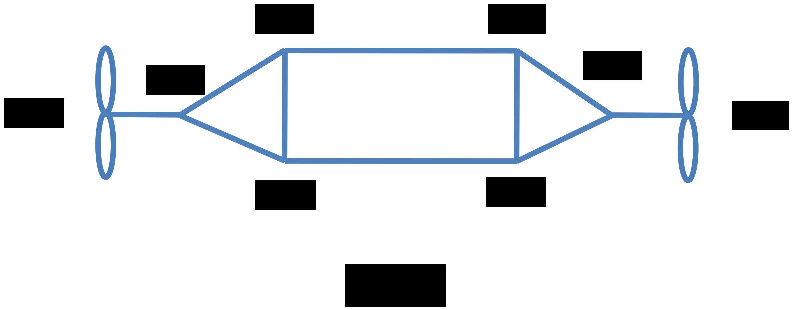
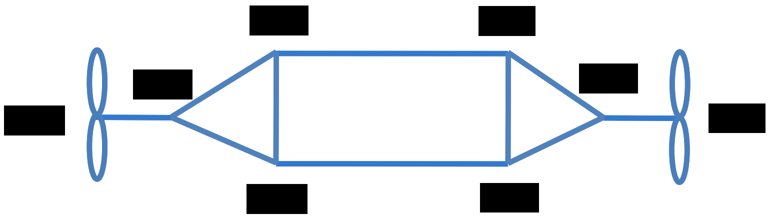


Decomposing SE

- Removal of „exchange edges“





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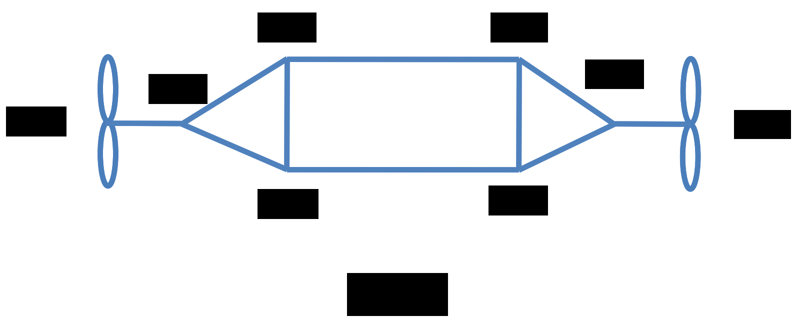

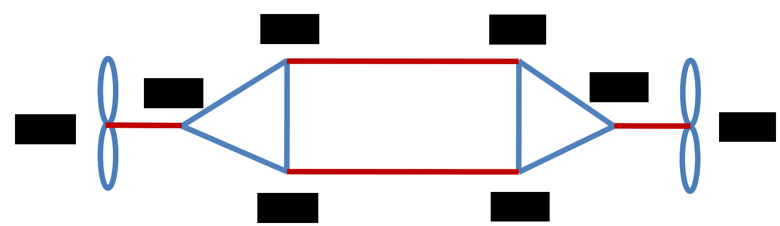

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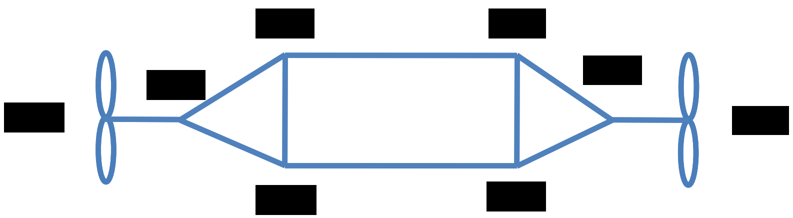

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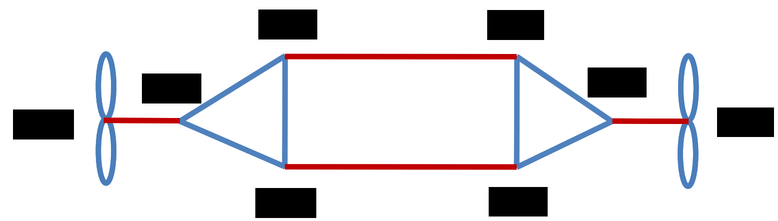

Similarity transform with

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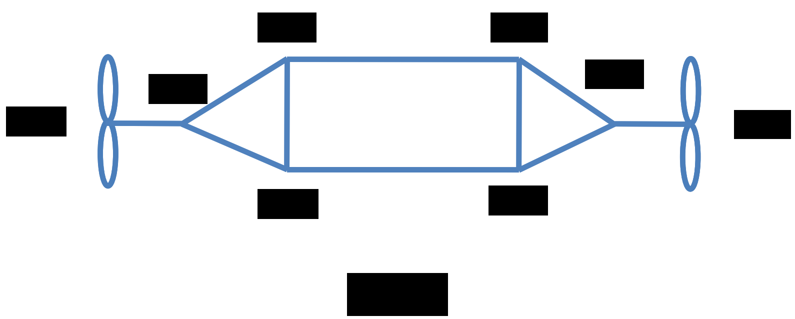

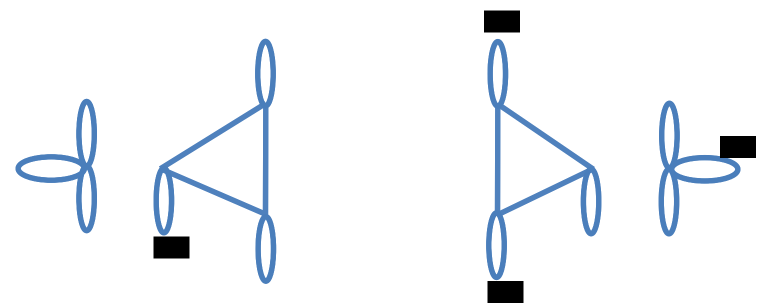

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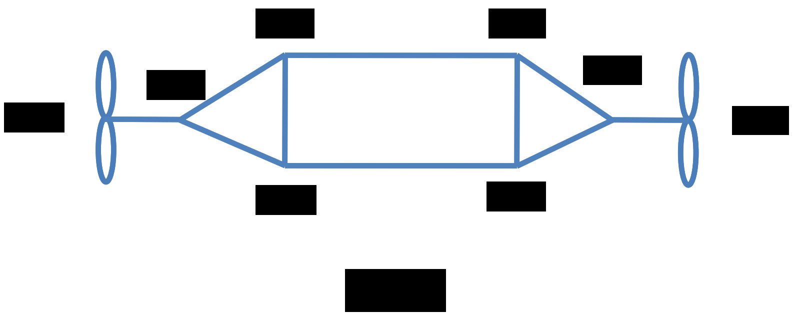

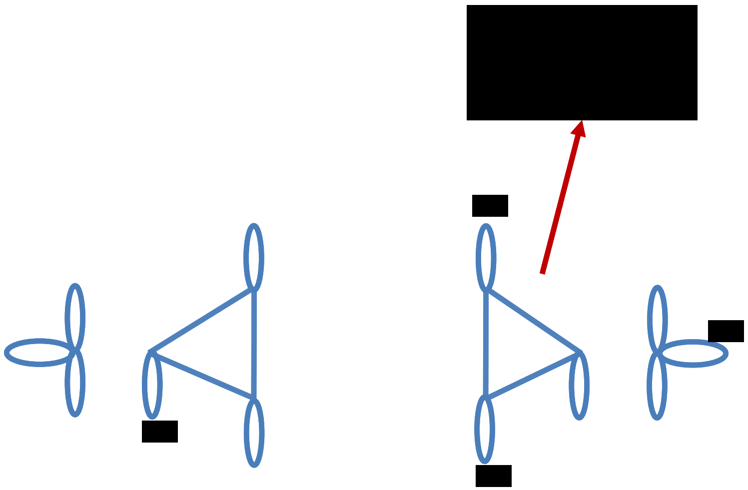

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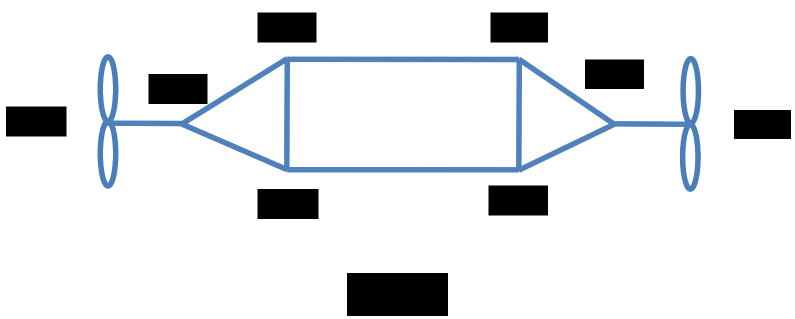

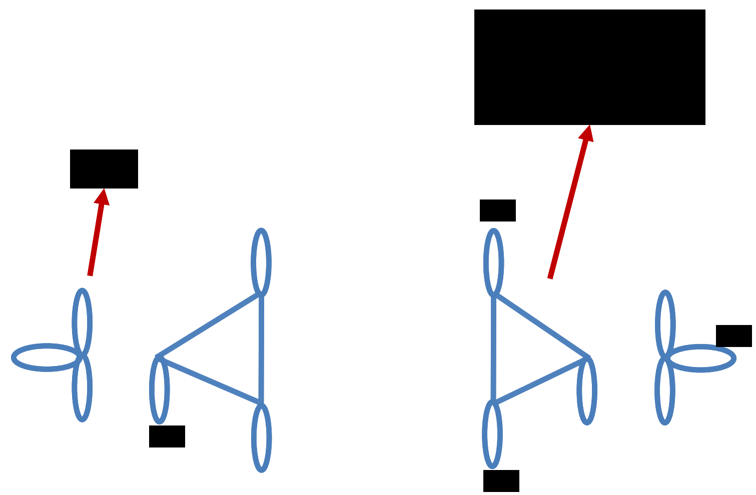

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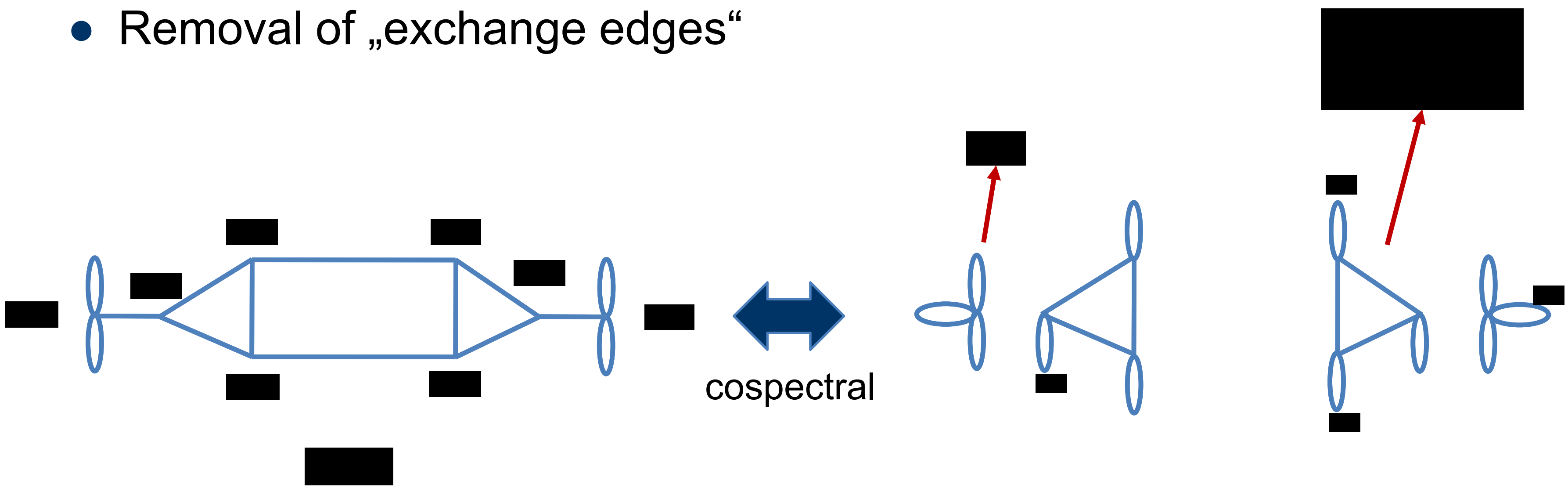

Similarity transform with

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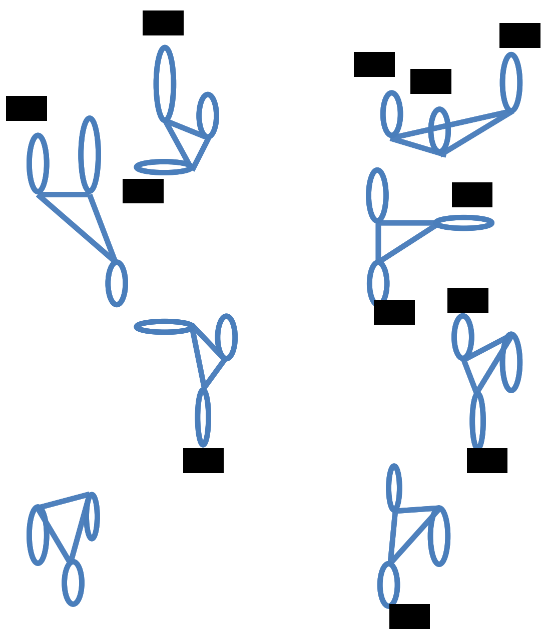
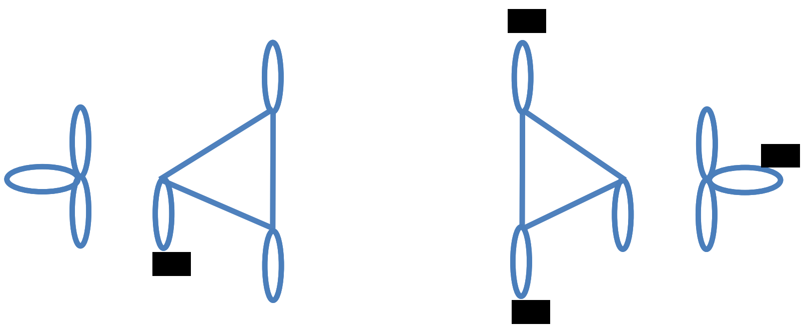
Relating the Circles

- We characterize the mappings [redacted] i.e.



Cospectral graph to [redacted]



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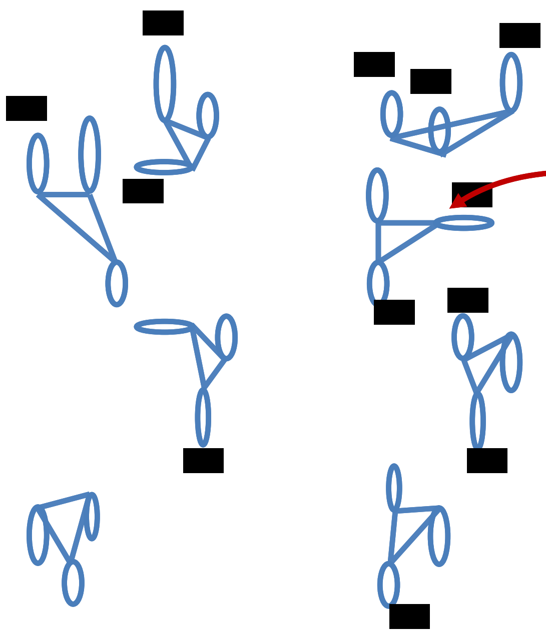

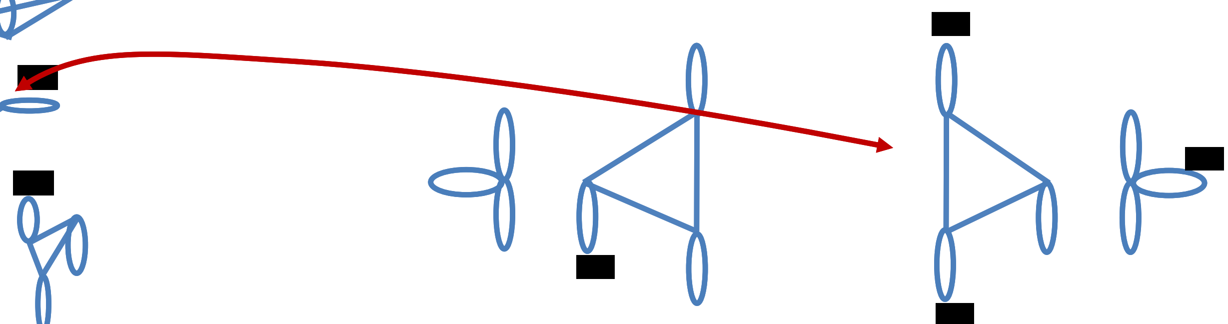

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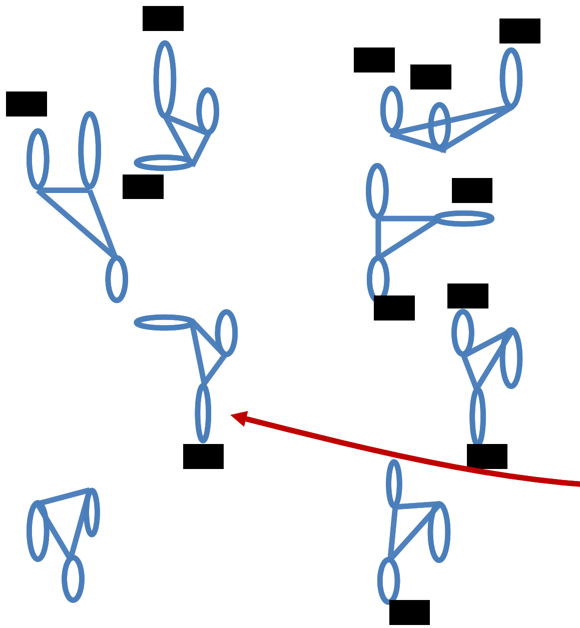

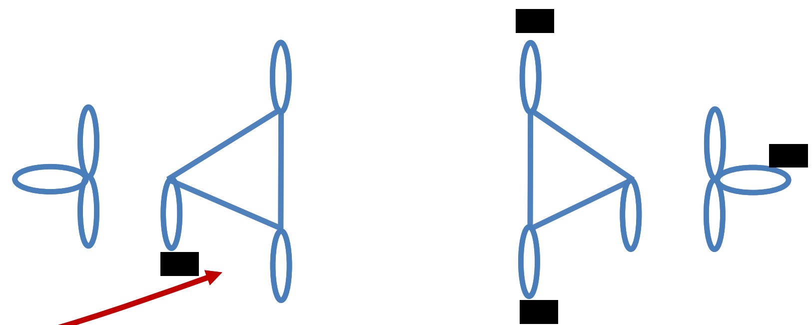

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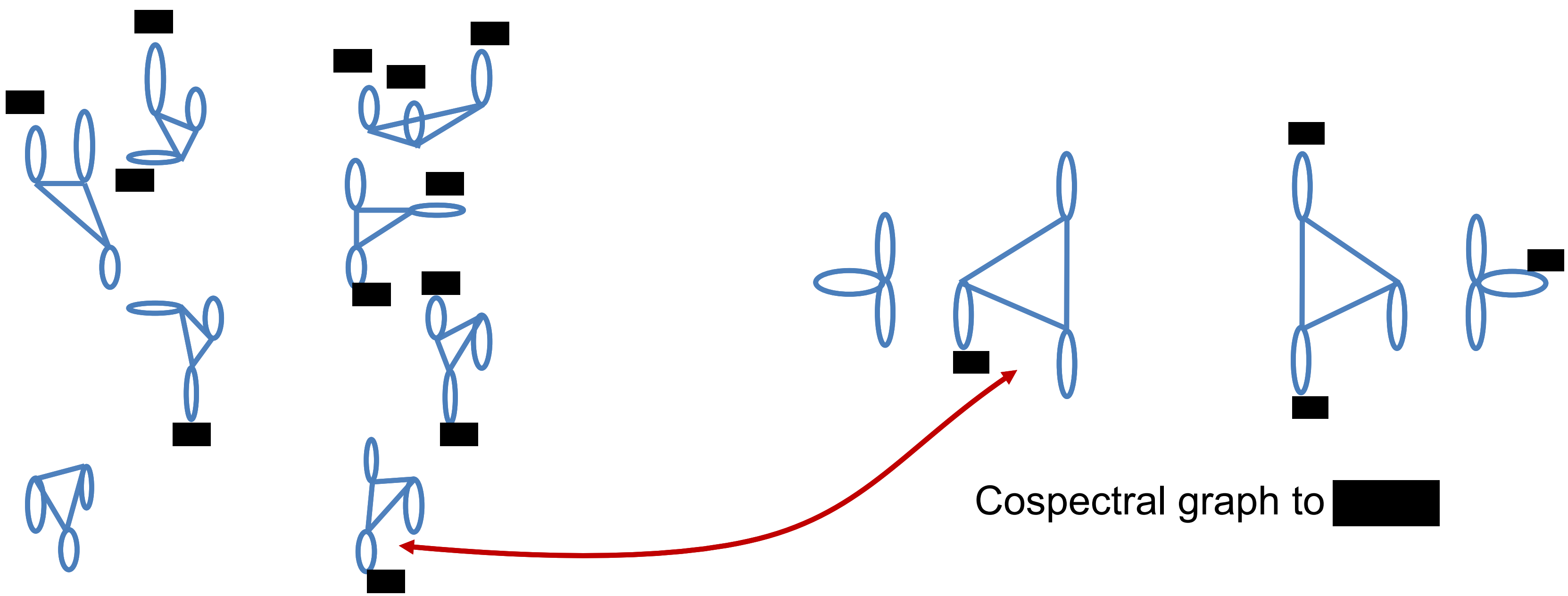

Cospectral graph to [redacted]

Cospectral graph to [redacted]

Result

- If [redacted] odd: [redacted]

- If [redacted] even: [redacted]

  - [redacted]

  - Note that [redacted] is not the only difference.

    We illustrate that in a second.

Result

There is always a complete eigenvalue mapping ☺

- If [redacted] odd: [redacted]
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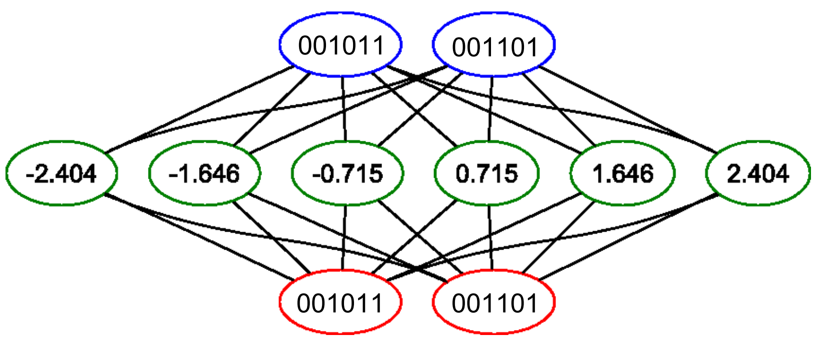
There is always a complete eigenvalue mapping ☺

There is never a complete eigenvalue mapping ☹

Relating the Circles

- Let [redacted]
- Denote e.g. $\begin{matrix} -1 & & \\ & -1 & \\ & & 1 \end{matrix}$ as 110 (by the diagonal entries)
- Aperiodic case: a cycle appears in both graphs



- Sometimes, this does not work!
  [redacted] compresses periodic cycles

100000	100000
100100	100
101010	10
111111	1

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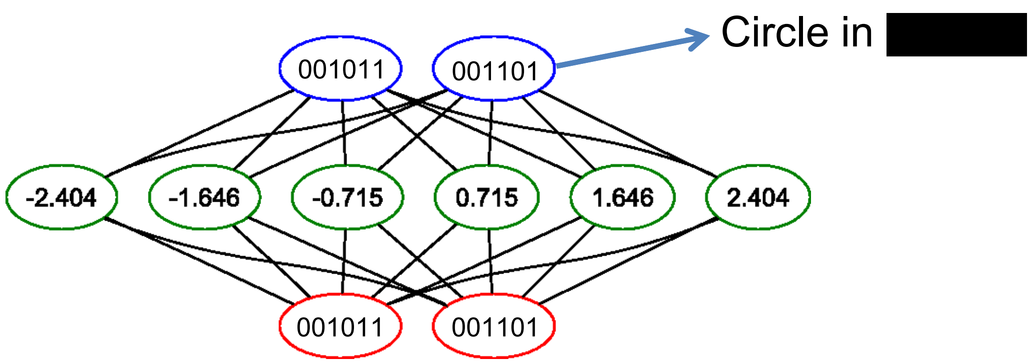

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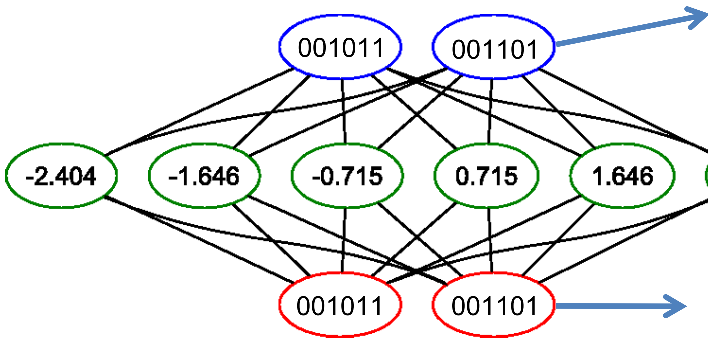

Circle in [redacted]

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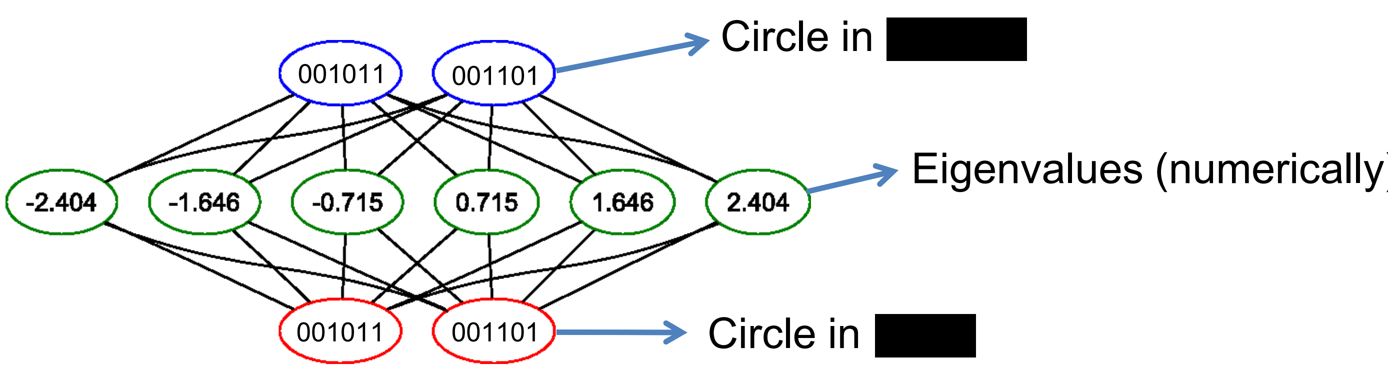

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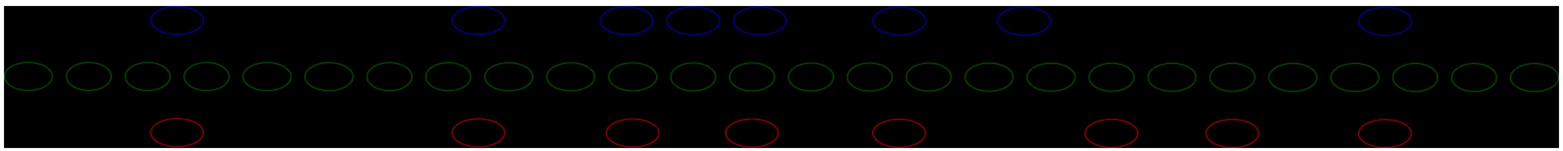
Non-trivial correspondences between EVs and Circles



011011 111111 000000 000001 001001

011 0 000001 001 111111

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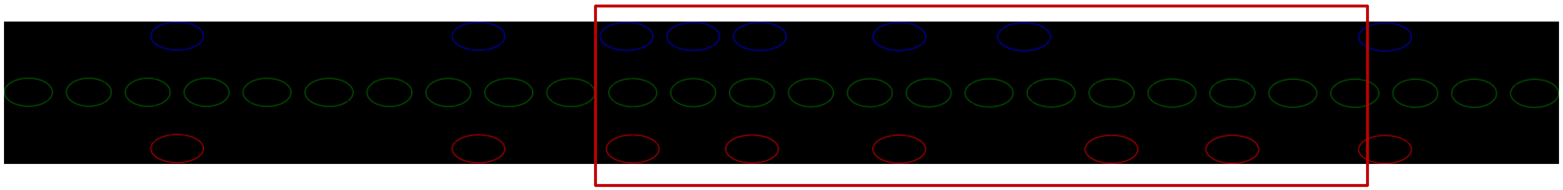

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011 0 000001 001 111111

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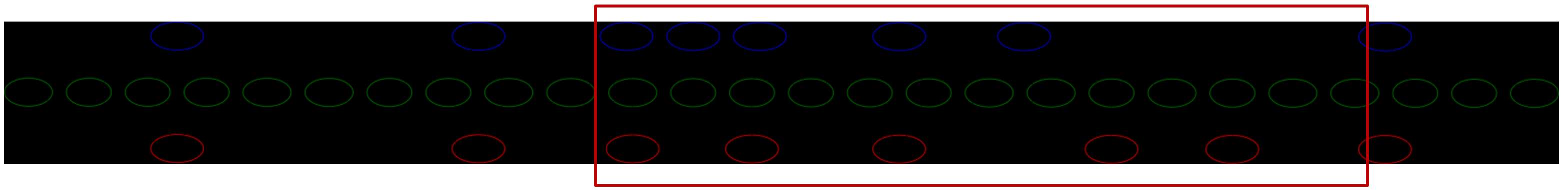

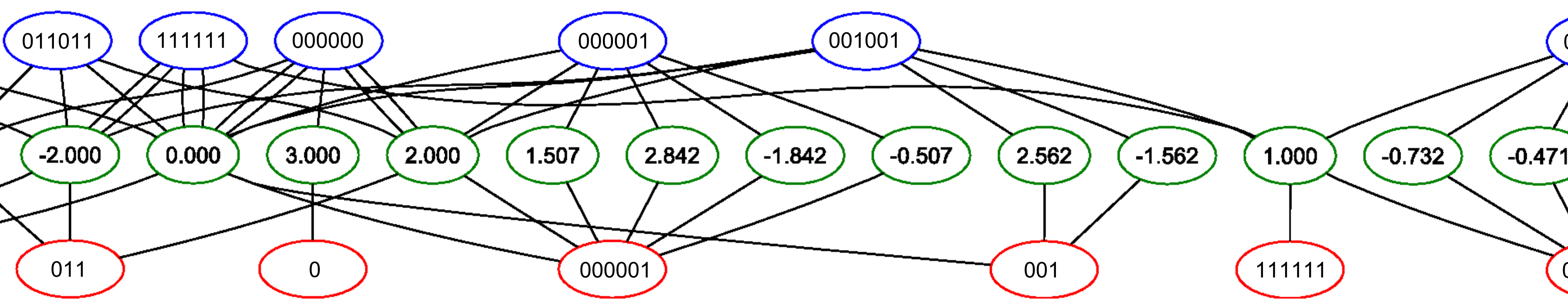

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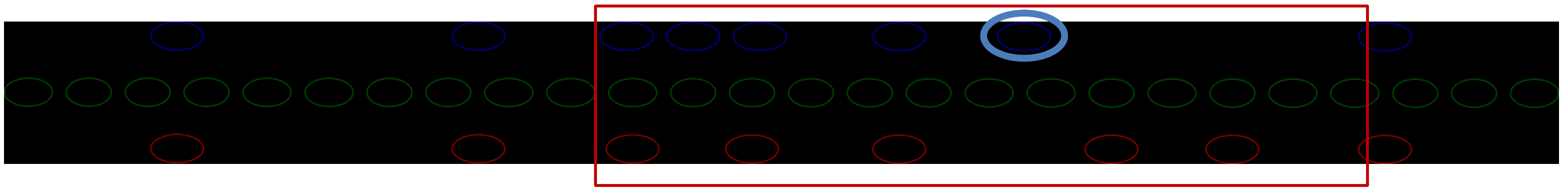

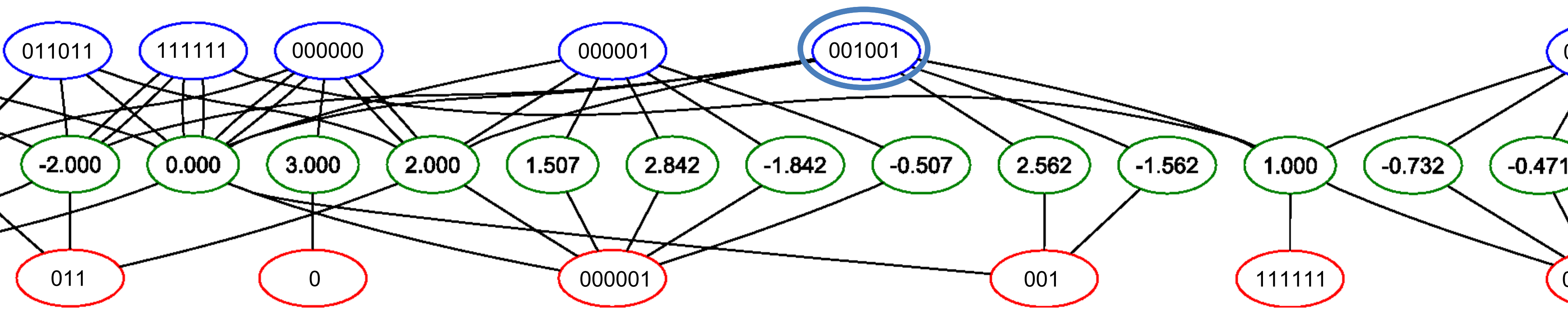

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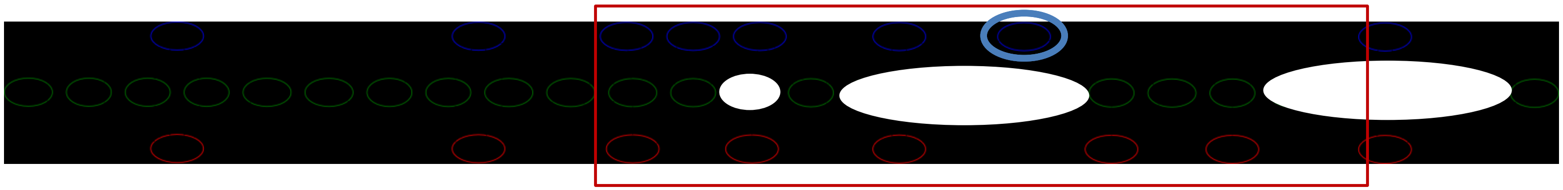

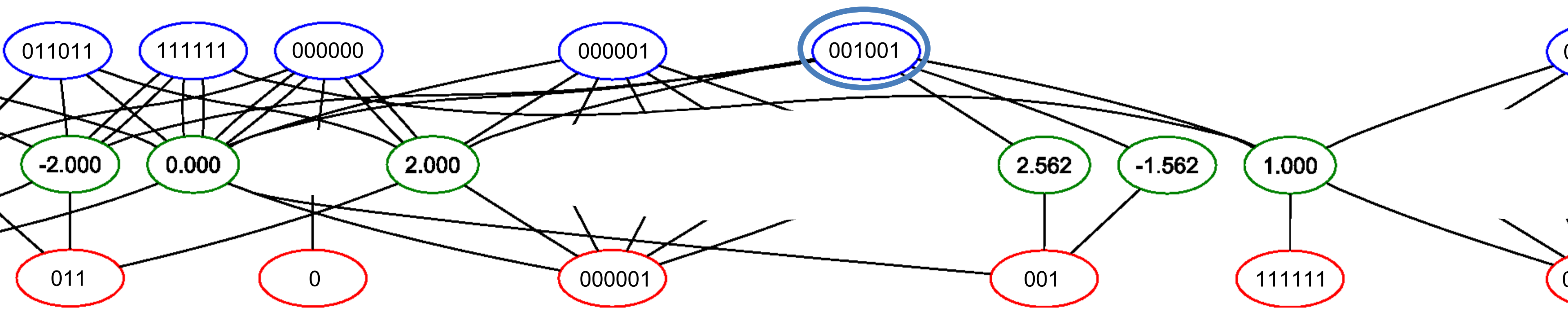

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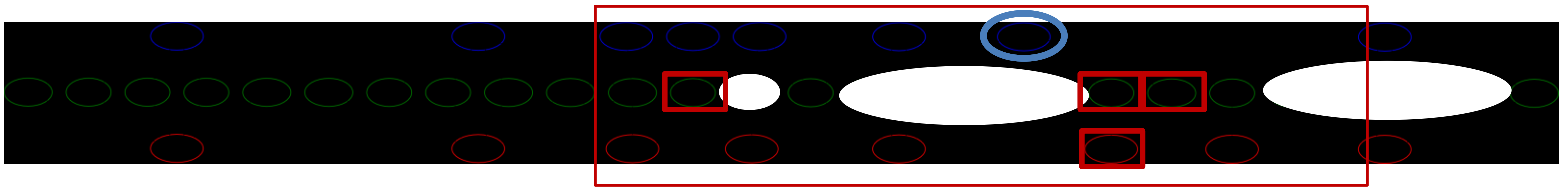

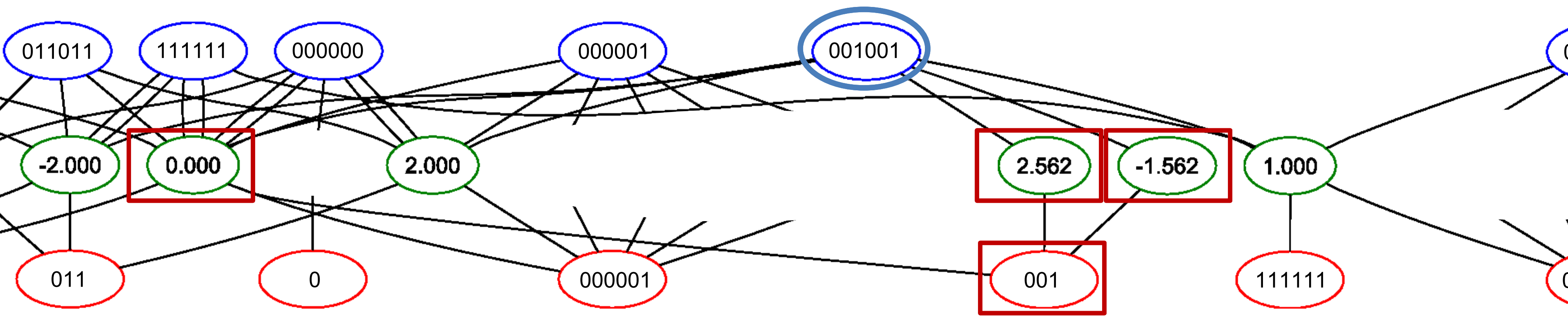

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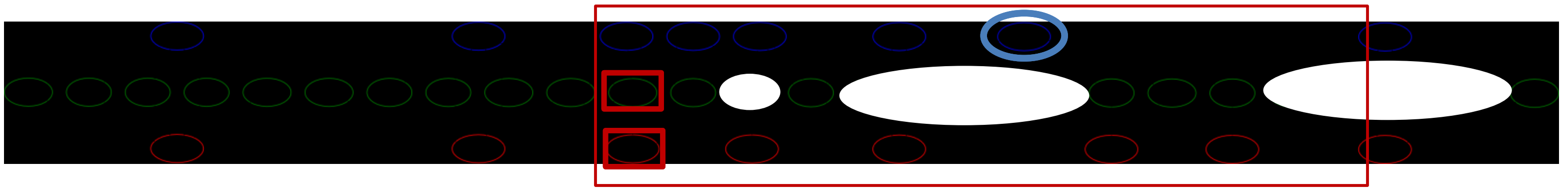

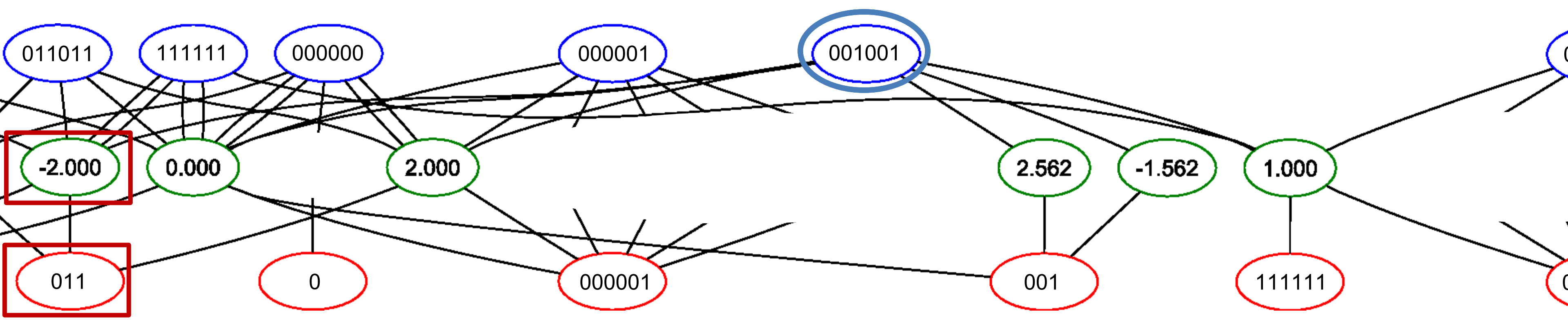

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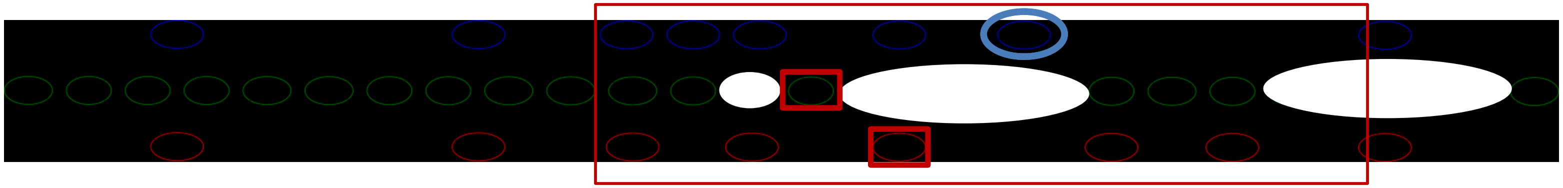

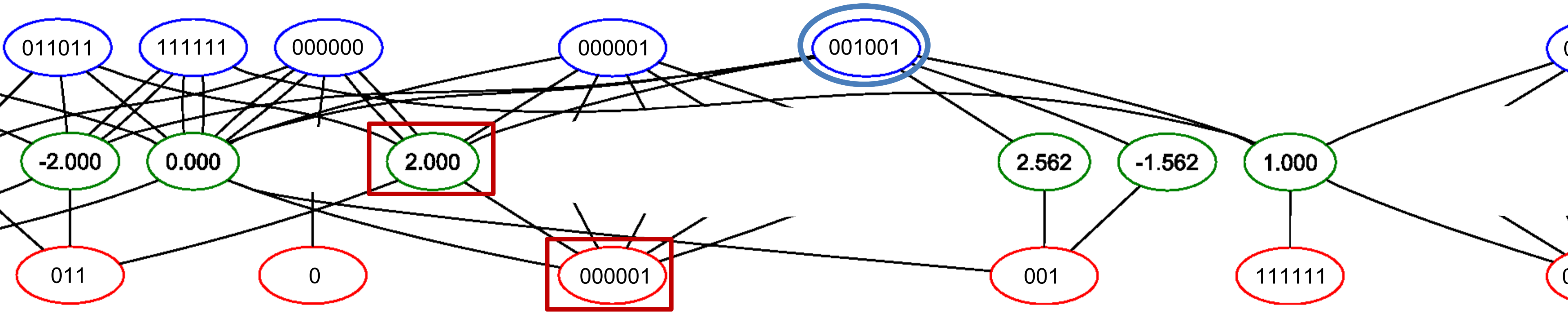

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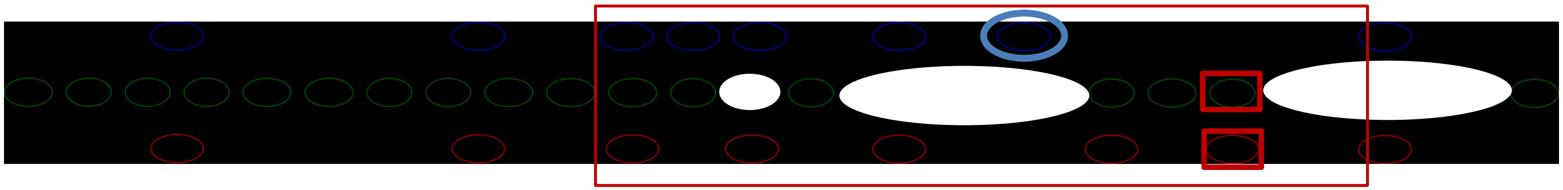

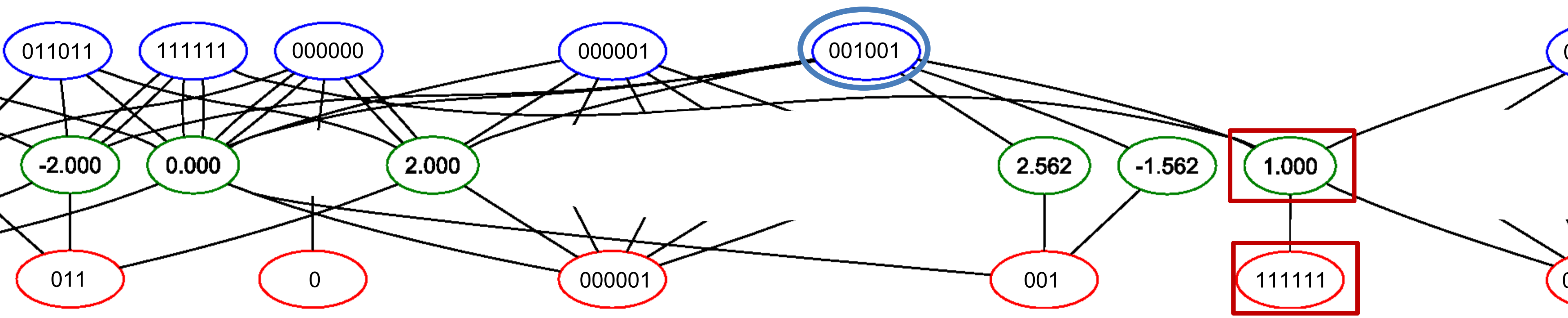

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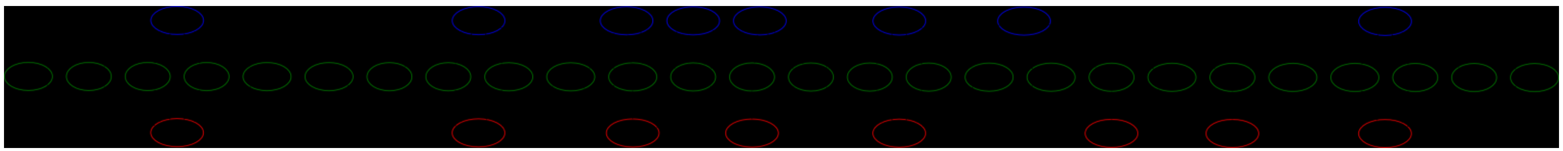

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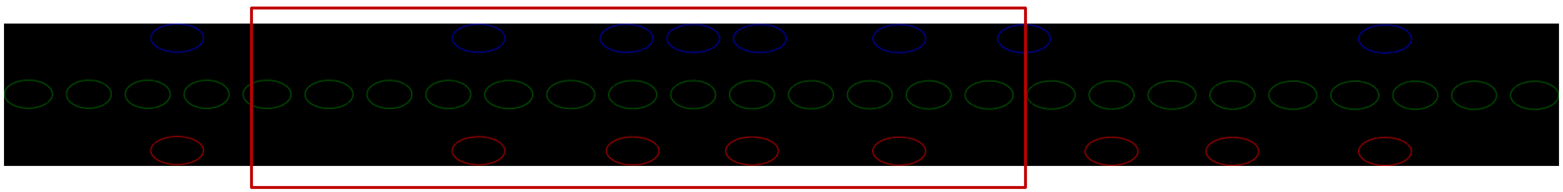

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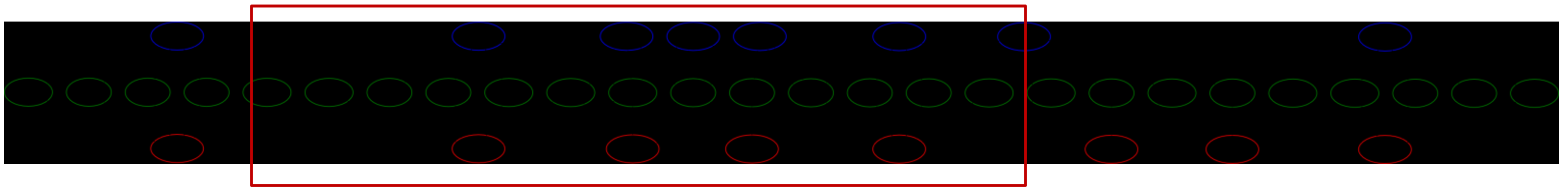

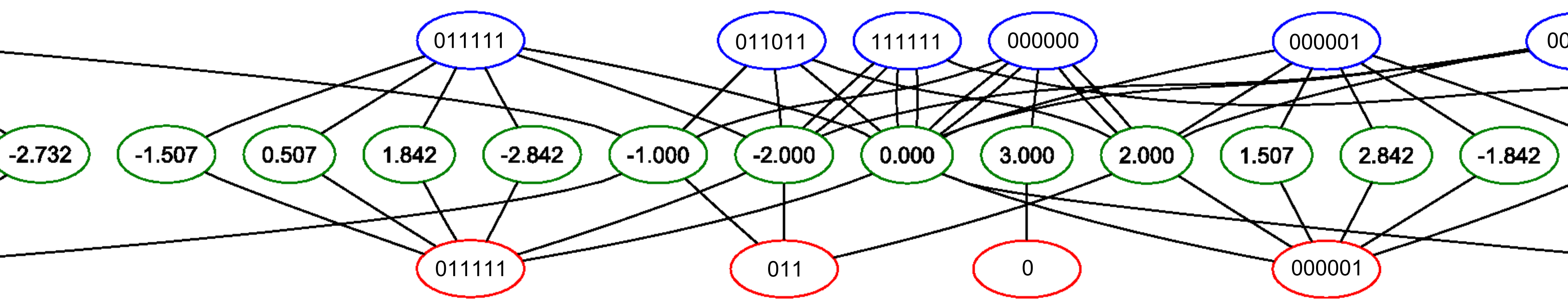

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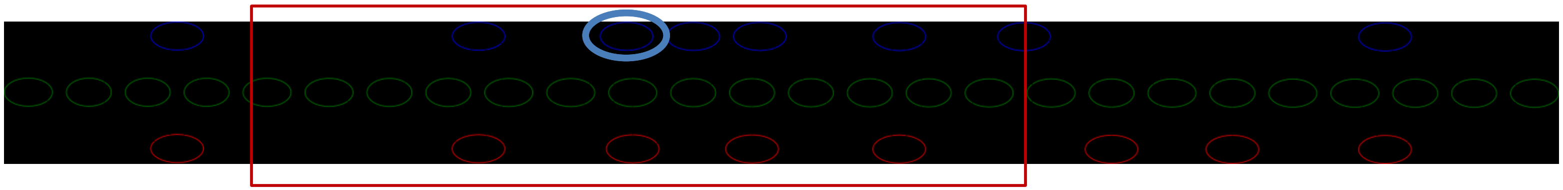

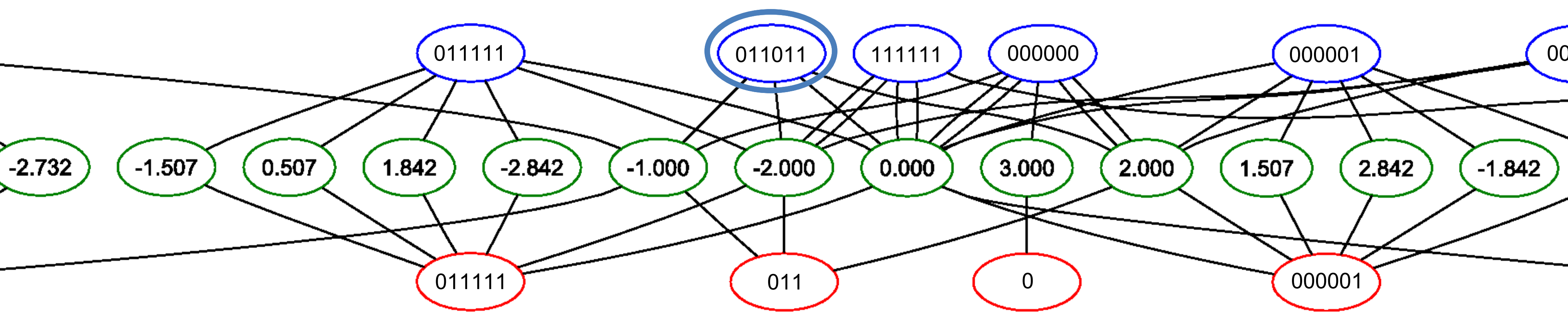

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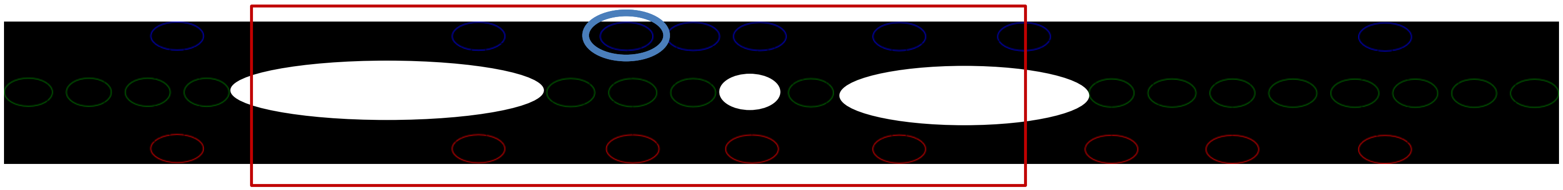

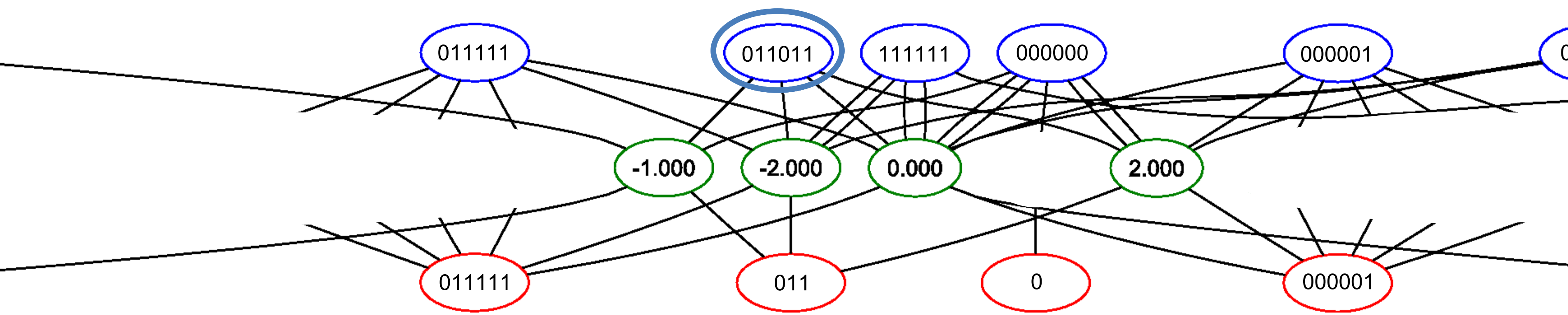

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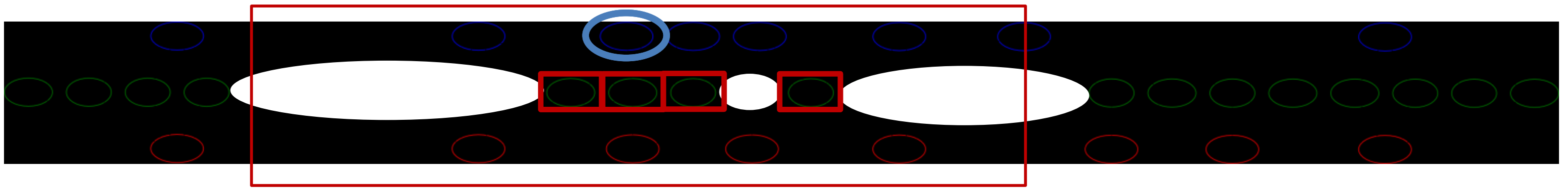

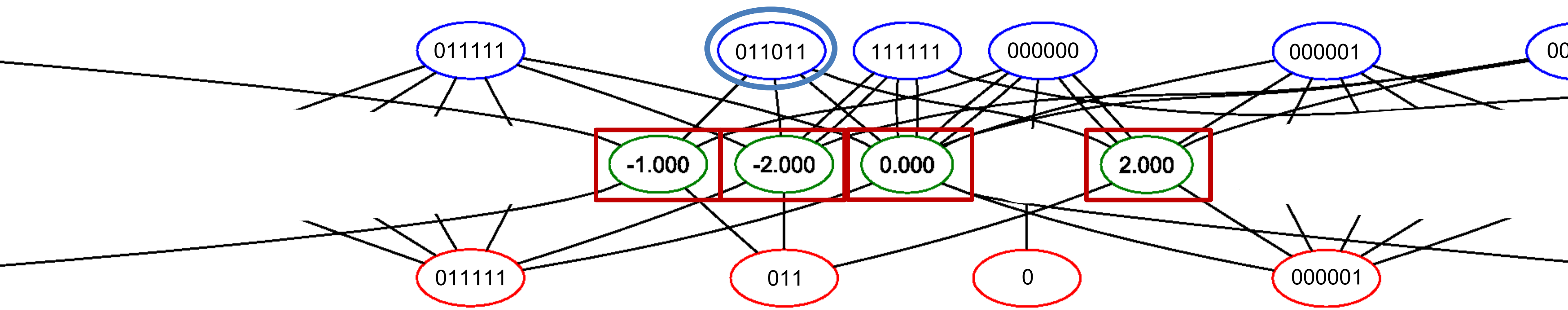

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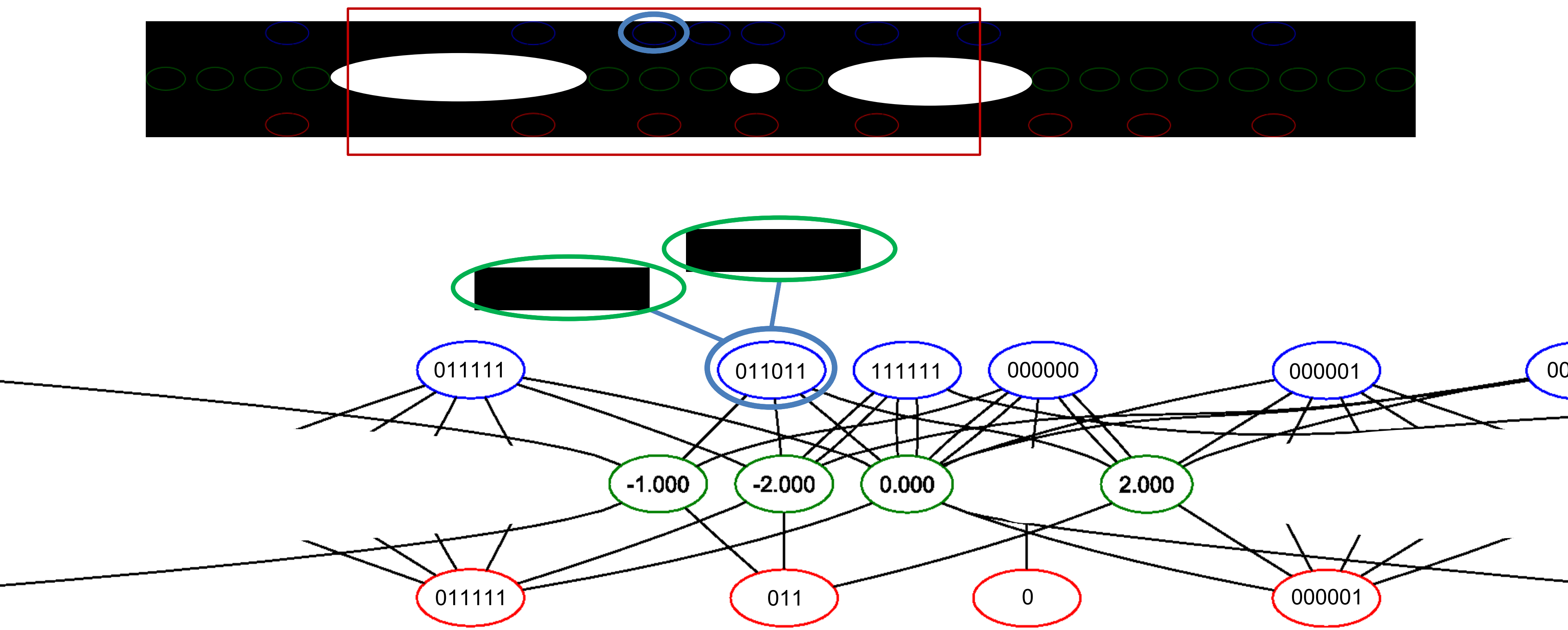

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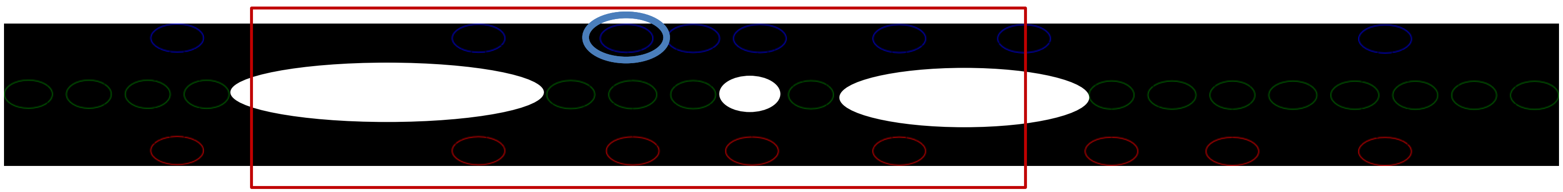

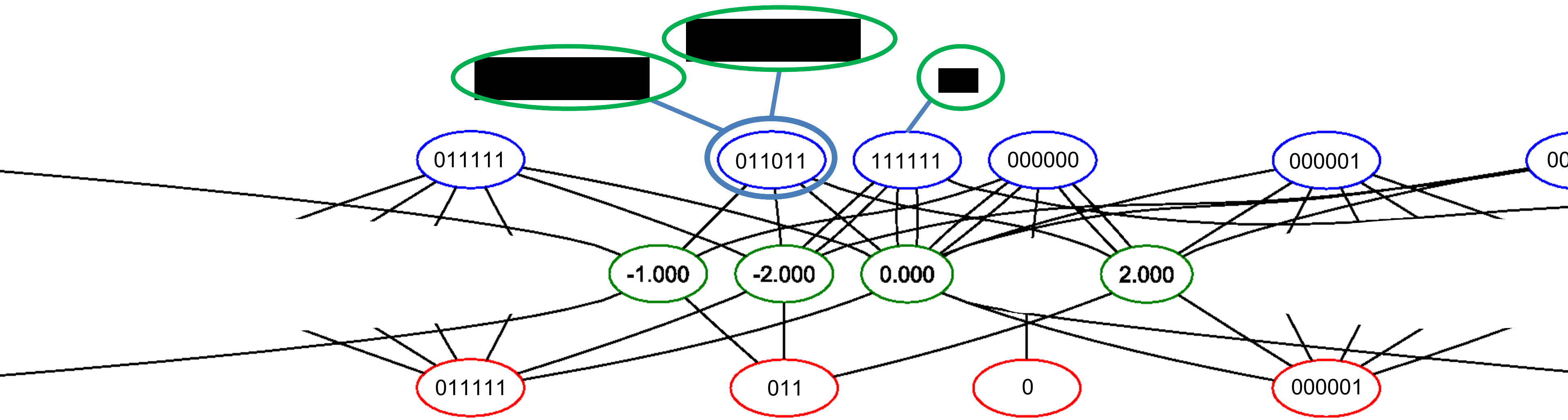

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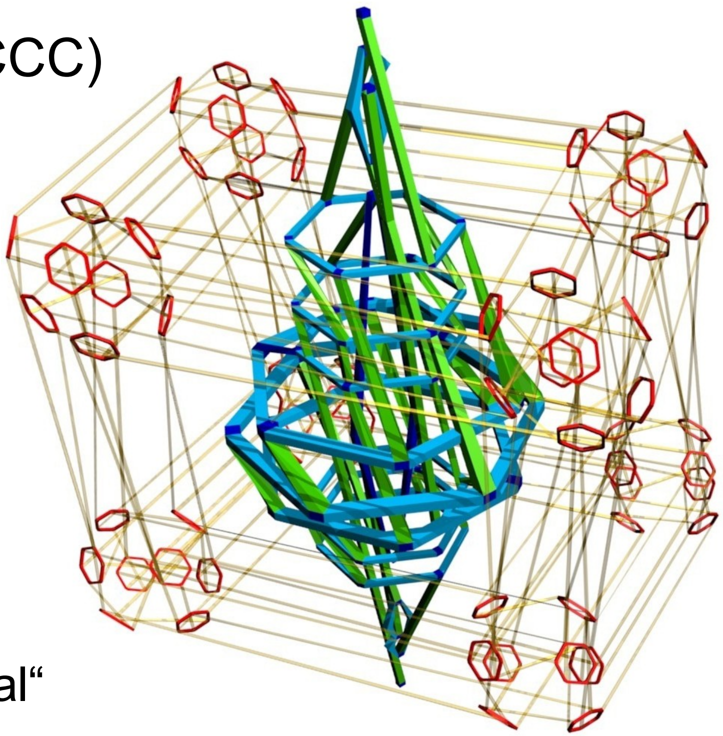

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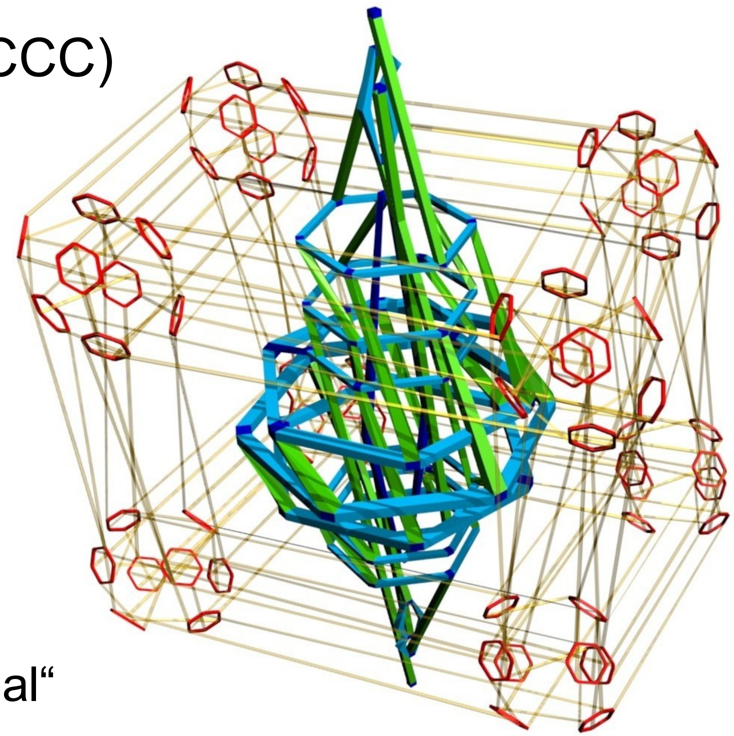

[redacted]

- Future work: pin down the “accidental”

Thank you for your attention.

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UNIVERSITÄT
ERLANGEN-NÜRNBERG

TECHNISCHE FAKULTÄT